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2023
Applied Mathematical Sciences
https://repository.ub.edu.bi/handle/123456789/1295

Applied Mathematical Sciences, Vol. 17, 2023, no. 13, 595 - 614
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ams.2023.917479

The FDP^{++}R$_{2023A}$ Performance Improvement Algorithm of Burundi Management System. The SATINOV Sciences in Africa

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Abstract

The statistical modeling of the problem of poverty is a major challenge given Burundi's economic development managment system. Innovative economic optimization systems are widely needed to face the problem of the dynamic of the poverty in Burundi. The Burundian economy shows an inflation rate of1.5% in 2018 for the Gross Domestic Product growth real rate of 2.8% in 2016.

This research concerns the FDP^{++}R$_{2023A}$ Algorithm for Performance Improvment of Burundi management system. The SATINOV management sciences and decision support in Africa. The aim is to find a management system model that contributes to solving the problem of poverty in Burundi. This model solves an optimization problem combining the variables of production, consumption, budget, human resources and available raw materials.

Scientific modeling and optimal solving of the poverty problem show the tools for measuring poverty rate and determining various countries' poverty levels when considering advanced knowledge for performance improvment of Burundi management system.

Keywords: Poverty Problem, Applied Statistic Modeling, Operational Research, Predictive algorithm, performance improvment, Burundi management system, Dynamic Programming, Matlab and Simulink, AMPL, KNITRO

Introduction

The government of Burundi **"Reta Nkozi, Reta Mvyeyi"** has made its Priority the eradication of poverty. All Burundi must have in a dignified and eternal manner **"Inoti y'ibihumbi cumi "10,000 BIF" mu mufuko"**. ISTA wants to make its contribution in the resolution and scientific mastery of the problem of poverty in Burundi.
In this research, it is about Dynamic Poverty Predictive algorithm for performance improvment of Burundi management system. Concretely, it is a question of modeling Burundi managment system by identifying the variables of poverty and their scientfical interactions by modeling the dynamics of the problem of poverty in Burundi, the objective function National efficiency in view of all its variables and national potential and finally the production and consumption constraints of all layers of the Burundian population.
The main problem of this research is to develop a scientific management system model for performance improvment and plan well the economic future of Burundi.
The Research hypotheses are:The current statistical and economic models are far from adequate to face the poverty and the exponential population; The development of a scientific model of the problem of poverty in view of the available resources is essential for a good mastery of the indicators of aid to decision at the micro and macroeconomic level.
The objectives of the projects are: To develop a more complete scientific model of the problem of poverty in view of the resources available in Burundi for a mastery of indicators of aid to poverty decision at the micro and macroeconomic level. To developp tools for planning well the economic future of Burundi; To Establish a normative component providing a series of indicators for those responsible for the fight against poverty to enable them to better understand the economic, administrative and organizational stakes in the fight against poverty; To support the choice of economic and political decision-makers within the framework of interventions in favor of Burundi; To Modelize and Plan the Process of Transfer and Incubation of Research Results.
For the mathematical modeling of the problem of poverty in Burundi, it is concretely to work on: Identification of Variables, Modeling of the dynamics of poverty , Constraints Modeling, Objective Function Modeling, Modeling of the Optimal Poverty Control Problem, Resolution digital by an Algorithmic implementation under a Matlab programming language with global optimization toolboxes Simulink, the Demonstration of algorithmic convergence and commutation, Extraction of the solutions which are the indicators of decision on the quantity of production, consumption, monetary and non-monetary yield in relation to the variables of the problem identified including the technical variables of control and mastery poverty in Burundi.

1 Mathematical Modeling of the Poverty Problem in Burundi

The Mathematical Modeling of the Poverty Problem in Burundi mainly concerns the Identification of Variables, the Modeling of the dynamics of poverty, the Modeling constraints, the Modeling of the objective function and finally the combination of all these functions for a complete Modeling of the optimal poverty control problem.

1.1 Identification of governance state variables and governance technical control variables

We have identified variables according to the different categories of raw resources available across all areas of agricultural, industrial and artisanal production,health, education, governance and services . These are mainly the following resources, etc:

- **Raw materials variables** : Gold, Coltan, Nickel, Limestone, Aluminum, Silver, Zinc, Clay Sand, Rocks, Peat, Quartz, Monazite, Uranium, Diamond, etc.
- **Agricultural products**: Tea, Coffee, Mushrooms, Beans, Corn, Wheat, Sorghum, Eulesine, Palm, Bananas, Potato, Sweet potato, Cassava, Colocases, Soybeans , Cabbage, Carrots, Cotton, Tobacco, Pineapples, Mangoes, Oranges, Lemons, Papayas, Strawberries, Avocados, Pasta, Sunflower, Sugar cane, Rice, Peas, Peanuts, etc.
- **Livestock variables** : Cow Meat, Goat Meat, Pork Meat Sheep Meat, Rabbit Meat, Duck Meat, Chicken Meat, Fish, Eggs, Butters, etc.
- **Industrial variables**: Yoghurt, Cow cheese, Goat cheese, organo-mineral fertilizers, agricultural lime, Primus, Small Primus, Primus Nyongera, Amstel Blonde 65 cl, Amstel Royal, Amstel Bock, Amstel Blonde 50 cl, Lemonades, Wine, Kinju, Aquavie, Sangwe, Fungus, Akezamutima juice, Maracuja juice, Pineapple juice, cement, Mattress, Soap, Wheat flour, Corn flour, Corn flour Cassava, Sorghum flour, Alien flour, Sesame flour, Cotebu loincloths, Pona Pharmaceutical products, SIPHAR products, Bread, Nails, Baking iron, Metal bars , Tôles, etc.
- **Consumption variables**: Men, Women and Children...........
- **Health Variables:** Birth, death, childbirth, health units, illnesses, medicines, health personnel, etc...

- **Education Variables:** academic success, academic failure, repetition, school attendance, teaching staff, etc...
- **Control Variables:** capital, labor, consumption, the volume of financial elements, production, etc...
- **Justice Variables:** lawsuits, land disputes, commercial disputes, etc...
- **Finances Variables:** National budget, investment budget, taxes, duties, salaries, expenses, receipts, public markets, etc...
- **Managment Variables:** standing committees, functional departments, administrative files by domain, etc...
- **Service system variables**: education, health, livestock, agriculture, environment, forests, etc,
- **Territorial and land variables:** economic infrastructures,
- **Social variables:** victims, disabled, street children, vulnerable, destitute, etc...

This model considers animals as production variables. Mathematically, we have the state variables $y_1, ..., y_n$ which give a vector function $y = (y_1, ..., y_n)^T$, The Technical variables for poverty control problem $w = (w_1, ..., w_p)^T$ as well as their interaction for the complete modeling of the mathematical model.

1.2 Modeling the dynamics of poverty [1, 2, 3, 4, 5, 6]

The dynamics of poverty is a system of first-order, non-linear and non-convex differential equations, all the more so as the problem of poverty is complex and multi-variable with implications for consumption, depletion and regeneration of resources.

$$\frac{dy}{dt} = f(t, y_1(t), ..., y_n(t), w(t)) = FGT_\alpha = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{z - R_i}{z}\right)^\alpha I(R_i \leq z) \quad (1)$$

$$I(R_i \leq z) = 1 \text{ si } R_i \leq z \quad (2)$$

$$= 0 \text{ sinon} \quad (3)$$

when $\dot{y} = FGT_\alpha$, et $R_i = PC_r/N$, the income per individual. The component of the dynamics of poverty is modeled as follows:

$$\dot{y}_i = f(T_i, K_i, E_i, D_i) = m \times r_i \times T_i^a K_i^\beta E_i^\gamma D_i^S \quad (4)$$

While the Production function P is given by the following model:

$$P = \sum_{i=1}^{n} m \times r_i \times T_i^a K_i^\beta E_i^\gamma D_i^S \tag{5}$$

when m is a parameter, z is the poverty line , α is the aversion coefficient which will determine the significance of the index. If α is high, a large weight of the poor will be taken into account. $r = (r_1, \cdots, r_n)$ is the productivity , $T_i = \frac{y_i}{r_i}$ the work component , $K_i = \frac{y_i}{a'}$ is the component of capital where a' is the yield , $E_i = c'R_i + E_0$ with $c' = \frac{E_i}{R_i}$ the consumption component. The term C_r is the function of the reduced costs [3].Overall we have that $T(y_1, ..., y_n) = (T_1, ..., T_n)$ is the work with T_i which depends of $y_i, i = 1, ..., n.$, $K(y_1, ..., y_n) = (K_1, ..., K_n)$ is the Capital, $E(y_1, ..., y_n) = (E_1, ..., E_n)$ is the consumption, $D(y_1, ..., y_n, w) = (D_1, ..., D_n)$ represents the volume of financial items . The differential equation $f(t, y_1(t), ..., y_n(t), w(t))$ is the function defining the dynamics of poverty in Burundi. By mathematical transformation, we have

$$\begin{aligned} P &= \sum_{i=1}^{n} m \times r_i \times \left(\frac{y_i}{r_i}\right)^a \left(\frac{y_i}{a'}\right)^\beta \left(c'R_i + E_0\right)^\gamma D_i^S \\ &= \sum_{i=1}^{n} m \times r_i^{1-a} \, \frac{y_i^{a+\beta}}{a'^\beta} \left(c'R_i + E_0\right)^\gamma D_i^S. \end{aligned} \tag{6}$$

Considering the fluctuations of financial elements, their volume D_i can be expressed by the following stochastic differential equation:

$$\begin{aligned} dD_i(t) &= N_i(t)dc_i(t) + \mu_i(t)c_i(t)dt - E_i(t)dt \\ &= \{\tau(t)N_0(t)c_0(t) + [\lambda_i(t) + \mu_i(t)]N_i(t)c_i(t) - E_i(t)\}dt && (7) \\ &+ N_i(t)c_i(t)\sigma_i(t)dW(t) \\ &= \{\tau(t)D_i(t) + [\lambda_i(t) + \mu_i(t) - \tau(t)]u_i(t) - E_i(t)\}dt && (8) \\ &+ \sigma_i(t)u_i(t)dW(t), && (9) \\ \dot{D}_i &= \tau(t)D_i(t) + [\lambda_i(t) + \mu_i(t) - \tau(t)]u_i(t) - E_i(t) && (10) \\ &+ \sigma_i(t)u_i(t)\frac{W(t)}{dt} && (11) \\ &= q(t, \tau(t), \lambda(t), \mu_i(t), \sigma_i(t), u_i(t), D_i(t), E_i(t), W(t)) && (12) \end{aligned}$$

where $\tau(t)$, $\lambda(t)$, $\mu_i(t)$, $\sigma_i(t)$ et $u_i(t) = N_i(t)c_i(t),\ i = 0, \cdots, n$ are respectively the interest rate, the appreciation rate, the dividend rate, the volatility and the wealth of the investment on the $i^{\text{è}}$ variable.
The Derivation of the individual income function is modeled by replacing P by its value in the individual income R_i:

$$R_i = \frac{\left[\sum_{i=1}^{n} m \times r_i^{1-a} \, \frac{y_i^{a+\beta}}{a'^\beta} \left(c'R_i + E_0\right)^\gamma D_i^S\right] C_r}{N}. \tag{13}$$

The technical control variables $w(t)$ are given by

$$\begin{aligned} w(t) &= (w_1(t), w_2(t), w_3(t), w_4(t), w_5(t), w_6(t)) \\ &= (P(t), T(t), K(t), E(t), D(t), C(t)) \end{aligned} \tag{14}$$

By injecting all these equations into (3), the real dynamic therefore takes the following form:

$$\begin{aligned} \dot{D}_i &= q(t, \tau(t), \lambda(t), \mu_i(t), \sigma_i(t), u_i(t), D_i(t), E_i(t), W(t)) \\ &= \tau(t) D_i(t) + [\lambda_i(t) + \mu_i(t) - \tau(t)] u_i(t) - E_i(t) + \sigma_i(t) u_i(t) \frac{dW(t)}{dt} \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{y} &= \frac{1}{N} \sum_{i=1}^{N} \left(\frac{z - \frac{\left[\sum_{i=1}^{n} m \times r_i^{1-a} \frac{y_i^{a+\beta}}{a'^{\beta}} \left(c' R_i + E_0 \right)^{\gamma} D_i^S \right] C_r}{N}}{z} \right)^{\alpha} I(R_i \leq z) \\ &= \frac{1}{N^2} \sum_{i=1}^{N} \left(\frac{Nz - \left[\sum_{i=1}^{n} m \times r_i^{1-a} \frac{y_i^{a+\beta}}{a'^{\beta}} \left(c' R_i + E_0 \right)^{\gamma} D_i^S \right] C_r}{z} \right)^{\alpha} I(R_i \leq z) \end{aligned} \tag{16}$$

The next section shows the relationship between state variables and control variables with the required raw resources.

1.3 Constraints of the Poverty Problem in Burundi.

The constraints are parameters and equations defining the problem. By considering the mathematical and economic complexity of the poverty problem, the constraints have ramifications for consumption, resource availability, and production [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13]. The combination of the required primary resources, the available primary resources and the variables constitute the mixed constraints given by the following system:

$$\left\{ \begin{array}{l} a_{11}y_1 + a_{12}y_2 + a_{13}y_3 + a_{14}y_4 + \cdots + a_{1n}y_n \leq b_1 \\ a_{21}y_1 + a_{22}y_2 + a_{23}y_3 + a_{24}y_4 + \cdots + a_{2n}y_n \leq b_2 \\ a_{31}y_1 + a_{32}y_2 + a_{33}y_3 + a_{34}y_4 + \cdots + a_{3n}y_n \leq b_3 \\ \cdots \\ a_{m1}y_1 + a_{m2}y_2 + a_{m3}y_3 + a_{m4}y_4 + \cdots + a_{mn}y_n \leq b_m \\ y_i \geq 0 \end{array} \right. \tag{17}$$

where $f(t, y_1, \ldots, y_n, w)$ is the function defining the dynamics of poverty in Burundi in time and space, the a_{ij}, $i = 1$,..., m, $j = 1$,..., n, are the required primary resources and the b_i are the available primary resources. In practice, the constraint matrix will be modeled on the basis of data collected in the field in the pilot provinces and will follow a generalization of the problem throughout Burundi country.

1.4 Objective function

The cost function of the poverty problem is $J(y_1, \ldots, y_n, w)$. It means the national yield function in view of the primary resources sold and consumed. This function is mathematically modeled as follows:

$$\begin{aligned} &J(y_1(t), \cdots, y_n(t), w(t)) = c_{m1}(t)y_1(t) + \cdots + c_{mn}(t)y_n(t) \\ &c_{mi}(t) = \tfrac{(\lambda t)^m e^{-\lambda t}}{m!}, m = 1, \cdots, 4, i = 1, \cdots, n \end{aligned} \tag{18}$$

Where m is the cropping season index. This function will be submitted to the maximization operator by ruling on the national maximum yield. In other words, it is a question of modeling the budgetary function if we consider all the possible economic variants. The $C(t) = (c_1(t), \cdots, c_n(t))$ is the function of the costs of the available resources sold. This function is vectorial with n variables defined above, namely $r = (r_1, \cdots, r_n)$ productivity $T = (T_1, \cdots, T_n)$ work, $K = (K_1, \cdots, K_n)$ Capital, $E = (E_1, \cdots, E_n)$ consumption and $D = (D_1, \cdots, D_n)$ volume of financial items, which include, in addition to the profits, all the items of the operating accounts more or less rebellious to the decomposition in volume and price index (interest, indirect taxes , etc.). In the absence of objective bases, D can be calculated by admitting that it varies, for example, like the volume of all the factors of production.

1.5 Mathematical Modeling of Optimal Control of the Poverty Problem

The combination of all the above equations demonstrated and their mathematical transformation allow us to pose the problem. The Mathematical Modeling of the Optimal Control of the Poverty Problem is therefore an optimization problem written as follows:

$$\begin{cases} \max\limits_{\mathbf{y}\in\mathbf{Y}} J(y_1(t), ..., y_n(t), w(t)) \\ \dot{y} = f(t, y_1(t), ..., y_n(t), w(t)) \\ A(t, y_1(t), ..., y_n(t), w(t)) \leq b \\ y_i \geq 0 \end{cases} \tag{19}$$

In this system, the first equation is the objective function of the national yield applied to a maximal optimization operator, the second differential equation is the dynamics of poverty and finally the last equations represent the system of constraints involved in the model. In order to simplify the writings for algorithmic rewriting questions, the problem takes the following form:

$$\begin{cases} \max\limits_{\mathbf{y}\in\mathbf{Y}} J(y(t), w(t)) \\ \dot{y} = f(t, y(t), w(t)) \\ A(t, y(t), w(t)) \leq b \\ y_i \geq 0 \end{cases} \tag{20}$$

2 Algorithmic resolution approach and commutation

2.1 4th order Runge-Kutta algorithm

1. Given a time step h, a maximum number of iterations N and an initial condition $(t_0, y_{1,0}, y_{2,0}, ..., y_{m,0})$

2. For $0 \leq n \leq N$
   For $i = 1, 2, 3, ...m : k_{i,1} = hf_i(t_n, y_{1,n}, y_{2,n}, ..., y_{m,n})$

3. For $i = 1, 2, 3, ...m : k_{i,2} = hf_i(t_n + \dfrac{h}{2}, y_{1,n} + \dfrac{k_{1,1}}{2}, y_{2,n} + \dfrac{k_{2,1}}{2}, ..., y_{m,n} + \dfrac{k_{m,1}}{2})$

4. For $i = 1, 2, 3, ...m : k_{i,3} = hf_i(t_n + \dfrac{h}{2}, y_{1,n} + \dfrac{k_{1,2}}{2}, y_{2,n} + \dfrac{k_{2,2}}{2}, ..., y_{m,n} + \dfrac{k_{m,3}}{2})$

5. For $i = 1, 2, 3, ...m : k_{i,4} = hf_i(t_n + h, y_{1,n} + k_{1,3}, y_{2,n} + k_{2,3}, ..., y_{m,n} + k_{m,3})$

6. For $i = 1, 2, 3, ...m : y_{i,n+1} = y_{i,n} + \dfrac{1}{6}(k_{i,1} + 2k_{i,2} + 2k_{i,3} + k_{i,4})$

7. $t_{n+1} = t_n + h$

8. For $i = 1, 2, 3, ...m : A_a(y_{i,n}, w_{i,n}) \leq 0,$

9. $\max_{\mathbf{y} \in \mathbf{Y}} J(y_1, \cdots, y_n, w_n) = c_1 y_1 + \cdots + c_n y_n + rT^a K^\beta E^\gamma D^S$

10. Display t_{n+1} et $\overrightarrow{y}_{n+1}$, $J(\overrightarrow{y}, \overrightarrow{w})$.

11. Stop

2.2 The FDP^{++}R$_{2023A}$ Algorithm for Performance Improvment of Burundi management system

The above modeled problem is a nonlinear and complex optimization problem. Let state the Hamilton function $H(t, y, p, w) = p(t, y, w)q(t, \tau(t), \lambda(t), \mu_i(t)\sigma_i(t), u_i(t), D_i(t), E_i(t), W(t)) + p(t, y, w)f(t, y, w) - A_a(t, y, w)$.
To solve this problem, Runge Kutta combined with a symplectic partitioned methods within the framework of a dynamic programming are applied
FDP^{++}R$_{2023A}$ Algorithm
1. By discretizing the time $[t_0, t_f]$ in N step $h = t_{n+1} - t_n = \frac{t_f - t_0}{N}$,N is the

maximum number of iterations, for $0 \leq n \leq N$,

$$\begin{aligned}
&H_w(w_{ki}, y_{ki}, p_{ki}) = 0\\
&D_{i,n+1} = D_{i,n} + h\sum_{k=1}^{s} b_k \mathcal{H}_p(w_{n_k}, y_{n_k}), D_i(t_0) = D_{i,0},\\
&displaystyle D_{i,n_k} = D_{i,n} + h\textstyle\sum_{j=1}^{s} a_{kj}\mathcal{H}_p(w_{n_j}, y_{n_j}), k = 1, .., s\\
&y_{n+1} = y_n + h\sum_{k=1}^{s} b_k \mathcal{H}_p(w_{n_k}, y_{n_k}), y(t_0) = y_0,\\
&y_{n_k} = y_n + h\sum_{j=1}^{s} a_{kj}\mathcal{H}_p(w_{n_j}, y_{n_j}), k = 1, .., s\\
&p_{n+1} = p_n - h\sum_{k=1}^{s} \tilde{b}_k \mathcal{H}_y(p_{n_k}, y_{n_k}), p_N = \phi'(t_f),\\
&p_{n_k} = p_n - h\sum_{j=1}^{s} \tilde{a}_{kj}\mathcal{H}_y(p_{n_j}, y_{n_j}),\\
&\mathcal{H}(y_{n_k}, p_{n_k}) = H(\psi(y_{n_k}, p_{n_k}), y_{n_k}, p_{n_k}),\\
&\tilde{a}_{kj} := b_j - \frac{b_j}{b_k} a_{jk}, b_j = \tilde{b}_j, A_a(y_n, w_{n_k}) \leq 0,\\
&Display\ t_{n+1}, y_{n+1}, w_{n_k}, p_{n+1}.
\end{aligned} \tag{21}$$

2. Stop

In this algorithm, a digitization of the state variables and the co-state variables as well as the control variables emerges. Consider that the variable index $i = 1, ..., N$ and that n is this time the iterative index, the above system becomes explicitly:

$$\begin{aligned}
&H_{wi}(w_{ki}, y_{ki}, p_{ki}) = 0, i = 1, ..., N\\
&D_{i,n+1} = D_{i,n} + h\sum_{k=1}^{s} b_k \mathcal{H}_p(w_{n_k}, y_{n_k}), D_i(t_0) = D_{i,0}\\
&D_{i,n_k} = D_{i,n} + h\sum_{j=1}^{s} a_{kj}\mathcal{H}_p(w_{n_j}, y_{n_j}), k = 1, .., s\\
&y_{i,n+1} = y_{i,n} + h\sum_{k=1}^{s} b_k \mathcal{H}_p(w_{i,n_k}, y_{i,n_k}), y_i(t_0) = y_0, i = 1, ..., N\\
&y_{i,n_k} = y_{i,n} + h\sum_{j=1}^{s} a_{i,kj}\mathcal{H}_p(w_{i,n_j}, y_{i,n_j}), k = 1, .., s, i = 1, ..., N\\
&p_{i,n+1} = p_{i,n} - h\sum_{k=1}^{s} \tilde{b}_k \mathcal{H}_{yi}(p_{i,n_k}, y_{i,n_k}), p_{i,N} = \phi'(t_f), i = 1, ..., N\\
&p_{i,n_k} = p_{i,n} - h\sum_{j=1}^{s} \tilde{a}_{kj}\mathcal{H}_y(p_{i,n_j}, y_{i,n_j}), i = 1, ..., N\\
&\mathcal{H}(y_{i,n_k}, p_{i,n_k}) = H(\psi(y_{i,n_k}, p_{i,n_k}), y_{i,n_k}, p_{i,n_k}),\\
&\tilde{a}_{kj} := b_j - \frac{b_j}{b_k} a_{jk}, b_j = \tilde{b}_j, i = 1, ..., N, A_{i,a}(y_{i,n}, w_{i,n_k}) \leq 0, i = 1, ..., N\\
&Afficher\ t_{n+1}, y_{i,n+1}, w_{i,n_k}, p_{i,n+1}.
\end{aligned} \tag{22}$$

An AMPL programming language will be coupled with Gurobi, KNITRO SNOPT solvers for the extraction of solutions. The following section is devoted to the discussion and interpretation of the solutions as well as the optimization and convergence criteria.
Considering complexity constraints matrix, questionnaire survey of administrative real data is used in order to complete the optimal modeling problem. This administrative data concerns statistical stratifications of Burundian pilot province. The Data consist of the state and control variables representing a non analytical abstract optimal modeling of the poverty problem. Details of this Data are included in the numerical process programing.

3 Numerical output and convergence process

By using the following numerical scheme,we demonstrate the commutation of the two RK4 and SPRK4 numerical methods. The Figure 1 shows the solving

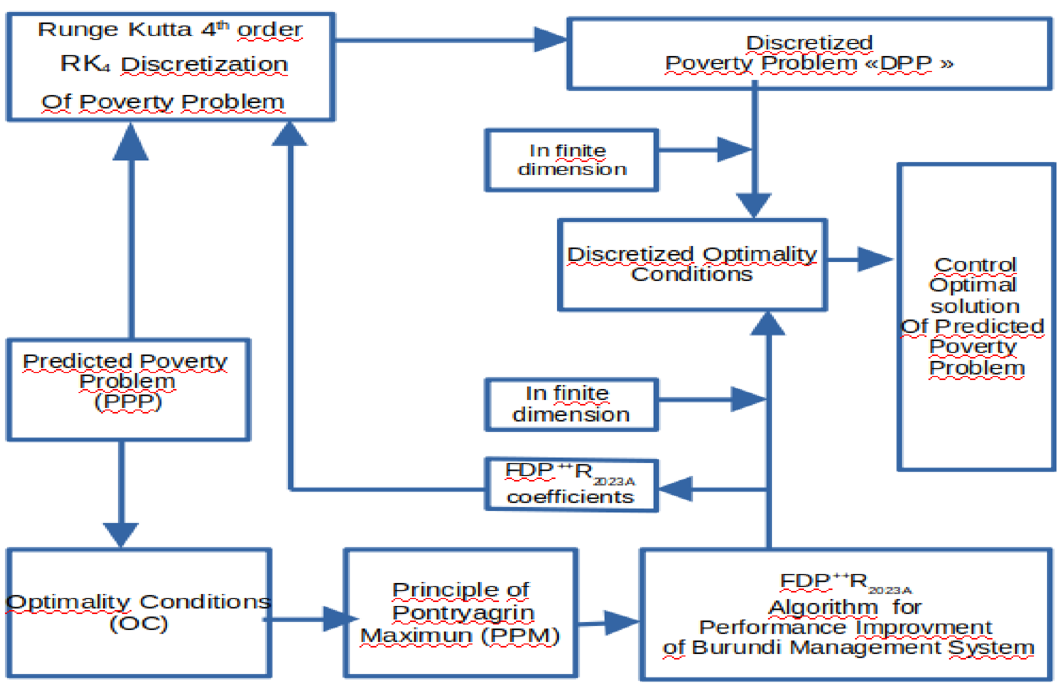

Figure 1: The Problem Solving Process: FDP^{++}R$_{2023A}$ Algorithm for Performance Improvment of Burundi management system.

method of this problem by demonstrating the Commutation Process of RK4 and SPRK4 Numerical Methods, to run through a robust algorithm implementation under an AMPL programming language A Streamlined Modeling for Real Optimization combined with KNITRO, Gurobi & SNOPT solvers.

The AMPL system is a sophisticated modeling tool that supports the entire optimization modeling lifecycle: development, testing, deployment, and maintenance. By using a high-level representation that represents optimization models in the same ways that people think about them, AMPL promotes rapid development and reliable results. AMPL integrates a modeling language for describing optimization data, variables, objectives, and constraints; a command language for browsing models and analyzing results; and a scripting language for gathering and manipulating data and for implementing iterative optimization schemes. All use the same concepts and syntax for streamlined application-building.
AMPL Powerful modeling language features: Broad support for sets and set operators. AMPL models can use sets of pairs, triples, and longer tuples, collections of sets indexed over sets; unordered, ordered, and circular sets of objects and sets of numbers; General and natural syntax for arithmetic, logical, and conditional expressions, familiar conventions for summations and other iterated operators; Automatic handling of linear and convex quadratic problems in continuous and integer variables; Nonlinear programming features such as initial primal and dual values, user-defined functions, fast automatic differentiation, and automatic elimination of "defined" variables; Convenient alternative notations for network flows, piecewise-linearities, complementarity conditions, and logical implications.
Valuable modeling support features: Interactive command environment with batch processing options. Powerful display commands let you view any model component or expression, browsing on-screen or writing to a file, using automatic formatting or your own preferences;Powerful scripting language including looping and if-then-else commands. Programs in the AMPL command language can define sophisticated iterative schemes that process input data, repeatedly adjust and solve instances of multiple models, and prepare results for analysis; Separation of model and data. AMPL models remain concise even as sets and data tables grow. Models may incorporate many kinds of conditions for validity of the data; Data input and output connections. Concise statements relate the model data and results to the contents of relational data tables.In order to demonstrate the switching and the relevance of the solution, an algorithmic implementation under a Matlab programming language with global optimization toolboxes & Simulink will also be carried out.

4 Analysis relationship for Consumption versus Production

The Daily intake varies according to age, physical expenditure and sex for a moderately active person, per day. The following table 1 shows the Burundi

Population Necessary Consumption Nutritive Value in Calories.

Population Type	Population size	Person Daily consumption (Ca)	Year Consumption (Ca)
Children:0-17	5905136	2700	5.8354553952e12
Man : 18-65 years	2956808	2500	2.70547932e12
Women: 18-65 years	3611520	2100	2.775814272e12
Old person: 65 years and over	357713	2000	2.61845916e11
Total	12831177	-	9.1436635152e12

Table 1: The Burundi Population Necessary Consumption Nutritive Value in Calories.

It shows the population size when considering the following categories and their nutritive value needs. Those categories are so very meaningfull and they are considered as Decision-maker indicators. By modeling the optimal control poverty problem, the population consumption needs are used to canalized poverty control variables with the production system. The Figure 2 shows the

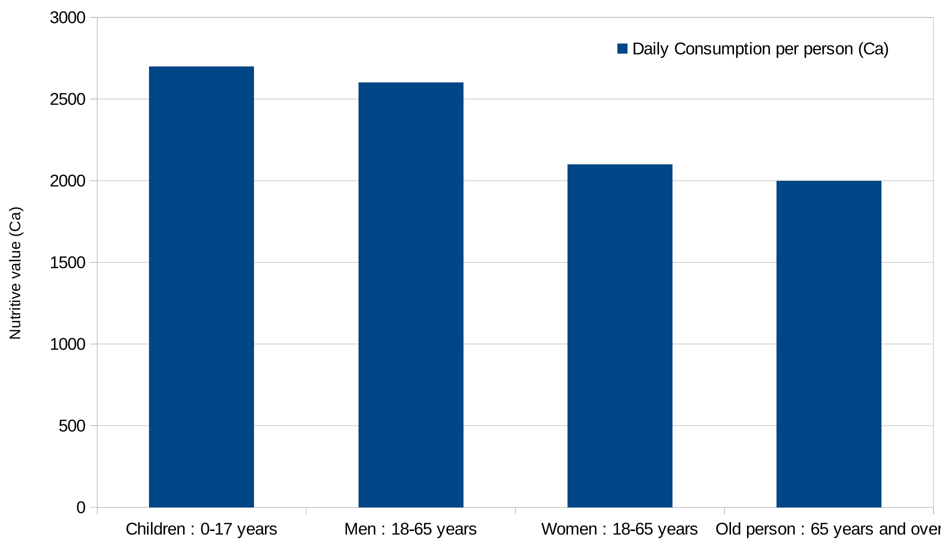

Figure 2: The Burundi Population Necessary Consumption Nutritive Value in Calories.

matching between population size categories and necessary Consumption Nutritive Value in Calories. The following Table 2 and Table 3 show relationship between population Necessary Consumption Nutritive Value in Calories and the national system Nutritive Value. In fact of eradicating poverty problem, state variables show quantity of production food when constraints applied are consumption, ressource availability and production.

Variables	**Year Production(Ton)**	**Production(Ca)/(100g)**	**Year Production(Ca)**
Fish	2.7706e4	0.208e3	5.762848e10
Vegetables	3.71892e5	0.33e3	1.2272436e12
Cucumber	0.18e4	0.13e2	2.34e8
squash	0.1875e4	0.30e2	5.625e8
zucchini	0.1875e4	0.30e2	5.625e8
Tea	1.1382e4	0.8e2	9.1056e9
Coffee	1.4058e4	0.1e4	1.4058e11
Mushrooms	0.393e3	0.28e2	1.1004e8
Bean	6.19151e5	0.12e3	7.429812e11
Maize	2.80813e5	0.356e3	9.9969428e12
Wheat	0.5628e4	0.149e3	8.38572e9
Sorghum	0.8851e4	0.328e3	2.903128e10
Eleusine	0.3084e4	0.151e3	4.65684e9
Palm	0.18e3	0.56e2	1.008e8
Bananas	1.179759e6	0.9e2	1.0617831e12
Potato	3.7644e5	0.9e2	3.38796e11
Sweet Potato	1.023458e6	0.11e3	1.1258038e12
Cassava	2.114848e6	0.14e3	2.9607862e12
Rice	2.52853e5	0.9e2	2.275677e11
Soja	1.2436e4	0.446e3	5.546456e10
Cabbage	4.4e4	0.28e2	1.232e10
Cauliflower	0.45e3	0.3e2	1.35e8
Onions	2.12e5	0.46e2	1.35e8
Leek	0.48e4	0.42e2	9.752e10
Carrots	0.45e4	0.38e2	1.71e9
Ananas	0.2e5	0.51e2	1.02e10
Mangoes	0.3e4	0.62e2	1.86e9

Variables	**Year Production(Ton)**	**Production(Ca)/(100g)**	**Year Production(Ca)**
Oranges	0.35e4	0.42e2	1.47e9
Lemons	0.28e4	0.4e2	1.12e9
Papayas	0.24e4	0.44e2	1.056e9
Strawberries	0.175e4	0.4e2	0.7e9
Avocados	3.6e5	0.2e3	7.2e11
Watermelons	0.42e4	0.3e2	1.26e9
Sunflower oil	1.469e4	0.9e3	1.3221e11
Peanut oil	1.469e4	0.9e3	1.3221e11
Palm oil	1.9216e4	0.56e2	1.076096e10
Sugar cane	2.18115e5	0.387e4	8.4410505e11
Small weight	1.2389e4	0.7e2	8.6723e9
Butters	0.1e5	0.76e3	7.6e10
Pepper	2.12e5	0.22e2	4.664e10
Tomatoes	8.75e4	0.2e2	1.75e10
Coconut	0.1e2	0.371e3	3.71e7
Guava	0.2e4	0.64e2	1.28e9
Cow meat	4.7071e4	0.25e3	1.176775e11
Yam	0.108e3	0.1e3	1.08e8
Goat meat	8.1456e4	0.3e3	2.44368e11
Pork meat	1.2655e5	0.3e3	3.7965e11
Mutton	6.8376e4	0.265e3	1.811964e11
Rabbit meat	0.15e4	0.16e3	2.4e9
Duck meat	7.03428e5	0.25e3	1.75875e12
Turkey	0.12e4	0.26e3	3.12e9
Pigeon	0.8e3	0.108e3	8.64e8
Chicken	8.1456e4	0.3e3	2.44368e11

Table 2: The National Production Nutrition Value in Calories (1).

The Table 4 and Table 5 characterize the food groups by their main nutrient intakes. They are called food constituents and their composition is different. All foods from a menu can be placed in any of them.

Variables	Year Production(Ton)	Production(Ca)/(100g)	Year Production(Ca)
Chicken Meat	8.1456e4	0.15e3	1.22184e11
Eggs	0.5e5	0.28e2	1.4e10
Yaourt	0.2e5	0.55e2	1.1e10
Cow cheese	0.2e5	0.8e2	1.6e10
Goat cheese	0.5e4	0.8e2	0.4e10
Primus Beer	9.45562e5	0.8e2	7.564496e11
Amstel Beer	9.45562e5	0.76e2	7.1862712e11
Lemonades	9.45562e5	0.37e2	3.4985794e11
Akezamutima juice	0.8e3	0.48e2	3.84e8
Maracuja juice	0.7e3	0.48e2	3.36e8
Ananas juice	0.8e3	0.54e2	4.32e8
Carotts	2.12e5	0.38e2	8.056e10
carotts juice	0.7e3	0.24e2	1.68e8
Céleri	0.48e4	0.2e2	9.6e8
Beetroot	0.4e4	0.4e2	1.6e9
Aubergine	0.35e4	0.29e2	1.4e9
Wheat flour	0.5628e4	0.364e3	2.048592e10
Maize flour	2.70813e5	0.37e3	1.0020081e12
Cassava flour	2.1665e5	0.35e3	7.58275e11
Bred	2.5e5	0.255e3	6.375e11
Sugar	2.0428e4	0.398e3	8.130344e10
Water	-	0	0

Table 3: The National Production Nutrition Value in Calories (2).

We show you the types of foods to control in your diet. These elements are meats, fish and eggs, Dairy products, fats, Cereals and Starches, Fruits and vegetables, Drinks and Sweet products. It is important to respect the balance between all these elements to enjoy their benefits, in bodybuilding or in all other sports.

Type of Food	Nutritious food composition	Ca/100g	Year production/100g	Year production by type of food (Ca)	Total by type of food
Proteins	Beans	16,64	6.19151e9	1.03026726400e11	1.2075834566e12
	Soja	153,87	1.2436e9	1.91352732e11	
	Cow cheese	6,4	0.2e9	1.28e9	
	Goat cheese	6,4	0.5e7	3.2e8	
	Maize	10,68	2.80813e9	2.99908284e10	
	cow meat	61,5	4.7071e8	2.8948665e10	
	Yam	1,69	1.08e8	1.8252e6	
	Patotoes	17,73	3.7644e9	6.6742812e10	
	Goat meat	143	8.1456e8	1.1648208e11	
	Pork meat	75	1.2655e9	9.49125e10	
	Mutton meat	88,51	6.8376e8	6.05195976e10	
	Rabbit meat	32,8	1.5e8	4.92e9	
	Duck meat	58,25	7.03428e9	4.0974681e11	
	Turkey meat	24,19	1.2e7	2.9028e8	
	Pigeon meat	25,812	0.8e7	2.06496e8	
	Chicken	76,20	8.1456e8	6.206472e10	
	Chicken meat	45,15	8.1456e8	3.6777384e10	

Table 4: The National Year Production By Type of food (Calories (1)).

Type of Food	Nutritious food composition	Ca/100g	Year production/100g	Year production by type of food (Ca)	Total by type of food
lipids	Butter	62,472	0.1e9	6.2472e9	2.6453448701e12
	Avocado	41,2	5.2776e10	2.1743712e12	
	Pork meat	31	1.2655e9	3.92305e10	
	Mutton meat	14,6	6.8376e10	9.982896e10	
	Rabbit meat	9,2	1.5e7	1.38e8	
	Peas	3,5	1.2389e8	4.33615e9	
	Duck meat	30,5	7.03428e9	2.1454554e11	
	Turkey meat	4,74	1.2e7	5.688e9	
	Pigeon meat	11.35	0.8e7	9.08e9	
	Chicken	10	8.1456e8	8.1456e9	
	Chicken meat	9,8	8.1456e8	7.982688e9	
	Cow meat	2,5	4.7071e8	1.176775e9	
	Fish	5.65	2.7706e8	1.565389e10	
	Palm oil	100	1.9216e8	1.9216e6	
	Peanut oil	96,7	1.469e8	1.420523e10	
	Sunflower oil	100	1.469e8	1.469e6	
	Corn	1,5	2.80813e9	4.212195e10	
	Bananas	0,35	1.179759e10	4.1291565e9	
	Corn	51	2.80813e9	1.4321463e11	
	Bananas	83	1.179759e10	9.7919997e10	
	Primus beer	23	9.45562e9	2.1747926e11	
	Amstel beer	23	9.45562e9	2.1747926e11	
	Ananas juice	139	0.8e7	1.112e10	
	Akezamutima juice	25	0.8e7	0.2e9	

Type of Food	Nutritious food composition	Ca/100g	Year production/100g	Year production by type of food (Ca)	Total by type of food
Minerals	Maracuja juice	8	0.7e7	5.6e8	5.9566475835e11
	Carrot juice	1,85	0.7e7	1.295e7	
	Coffee	15	1.4058e8	2.1087e9	
	Yogurt	7	0.5e9	3.5e9	
	Sweet potatoes	4	1.023458e10	4.093832e10	
Acids	Cow cheese	0,1	0.2e9	0.2e10	5.80712e9
	Goat cheese	0,4	0.5e8	0.2e9	
	Peas	0,2	1.2389e8	2.4778e9	
	Tomatoes	0,3	3.7644e9	1.12932e9	
Calcium	Cow cheese	78	0.2e9	1.56e10	6.385756e10
	Goat cheese	13	0.5e8	6.5e9	
	Eggs	9	0.5e9	4.5e9	
	Beans	6	6.19151e9	3.714906e10	
	Orange	0,031	0.35e8	0.1085e9	
Vitamins	Eggs	197	0.5e9	9.85e10	1.363975366e12
	Fish	254	2.7706e8	7.037324e10	
	Yogurt	56	0.2e9	1.12e10	
	Mangoes	91	0.3e8	2.73e9	
	Sweet potatoes	69	1.023458e10	7.0618602e11	
	Lemon	95	0.28e8	2.66e9	
	Orange	94	0.35e8	3.29e9	
	Ananas	44	0.2e9	8.8e9	
	Papayas	0,0653	0.24e8	1.56106e6	
	Avocado	127	3.6e9	4.572e11	
	Carrot	64	0.45e8	2.88e9	

Table 5: The National Year Production By Type of food (Calories (2)).

Type of Food	Nutritious food composition	Ca / 100g	Year production /100g	Year production by type of food (Ca)	Total by type of food
Fibers	Lemon	1,5	0.28e8	4.2e9	7.066505032e10
	Orange	1.5	0.35e8	5.25e9	
	Ananas	1,2	0.2e9	2.4e10	
	Papayas	1,3	0.24e8	3.12e9	
	Avocado	4	3.6e9	1.44e10	
	Soja	15,1	1.2436e8	1.877836e10	
	Small weight	5.8	1.2389e8	7.18562e8	
	Beans	3,2	6.19151e9	1.9812832e8	
Carbohydrates	Rice	28,8	2.52853e9	7.2821664e10	9.321735891e11
	Bread	44,3	2.5e9	1.1075e7	
	Carrot	9,58	0.45e9	4.311e9	
	Wheat flour	69.3	0.5628e8	3.900204e9	
	Maize flour	80	2.70813e9	2.166504e11	
	Cassava flour	35	2.1665e9	7.58275e10	
	Patatoes	16,2	3.7644e9	6.098328e10	
	Orange	8,03	0,35e8	2.8105e8	
	Leeks	4,9	0.48e8	2.352e8	
	Cabbage	4,8	4.4e8	2.112e9	
	Sugar	96,7	2.0428e8	1.9753876e10	
	Primus beer	1.6	9.45562e9	1.5128992e10	
	Amstel beer	4,6	9.45562e9	4.3495852e10	
	Ananas juice	13.1	0.8e7	1.048e8	
	Akezamutima juice	10	0.8e7	0.8e8	
	Maracuja juice	10.2	0.7e7	7.14e7	
	Carrot juice	9,58	0.7e7	6.706e7	
	Wheat	27,4	0.5628e8	1.542072e9	
	Sorghum	93.7	0.8851e8	8.293387e9	
	Beans	15.01	6.19151e9	9.29345651e10	
	Sweet potatoes	12,2	1.023458e10	1.24861876e11	
	Maize	67,2	2.80813e9	1.88706336e11	

Table 6: The National Year Production By Type of food (Calories (3)).

Population	Effectif	Proteins	Lipids	Minerals	Acids	Calcium	Vitamin	Fibers	Carbohydrates
Man : 18-65 years	2.956808e6	200	700	87	13	20	30	450	1000
Women : 18-65 years	3.61152e6	180	650	80	10	10	20	350	800
Children : 0-17 years	5.905136e6	300	800	87	13	20	30	450	100
Old person : 65 years and over	3.57713e5	180	600	80	10	10	20	350	750
Total per day	-	3.077364340e9	9.3559902e9	1.088527768e9	1.19415919e8	1.8442753e8	3.4524298e8	5.377106350e9	4.10452795e9
Total per year	-	1.12631534844e12	3.4242924132e12	3.98401163088e11	4.3706226354e10	6.750047598e10	1.2635893068e11	1.9680209241e12	1.5022572297e12

Table 7: The Necessary National Year consumption By Type of food (Ca).

Analysing Tables [4, 5,6,7], the comparing of the annual food production and the required annual consumption shows an imbalance between different types of food. Proteins (1.2075834566e12 Ca), minerals (5.9566475835e11 Ca) and vitamins (1.363975366e12 Ca) produced in Burundi are sufficient when considering their consumption being divided concretely into (1.12631534844e12 Ca) for proteins, (3.98401163088e11 Ca) for minerals , (1.2635893068e11 Ca) for vitamins as required by the entire Burundian population. This positive contribution for the latter comes from the fact that some cows, goats,fishes,..., slaughtered in Burundi come from neighboring countries. Real production remains in deficit. The lipids (2.6453448701e12 Ca), acids (5.80712e9 Ca), calcium (6.385756e10 Ca), fibers (7.066505032e10 Ca) and carbohydrates (9.321735891e11 Ca) produced in Burundi are insufficient for consumption. Indeed, their necessary annual consumption is distributed (3.4242924132e12 Ca) for lipids, (4.3706226354e10 Ca) for acids, (6.750047598e10 Ca) for fibers, (1.9680209241e12 Ca) for calciums and (1.5022572297e12 Ca) for carbohydrates. This negative contribution proves a Burundian food deficit. It is a decision-making indicator for the design and updating of agricultural policy and implementation programs and projects. Investment and economic growth are only possible when food security is mastered. The capital allocated to food investment must be revised upwards. Demographic control is also a relevant indicator to establish Burundi in the list of Emerging Countries in 2040 [6].

The Figure 3 shows the real simulation of necessary national year nutritive

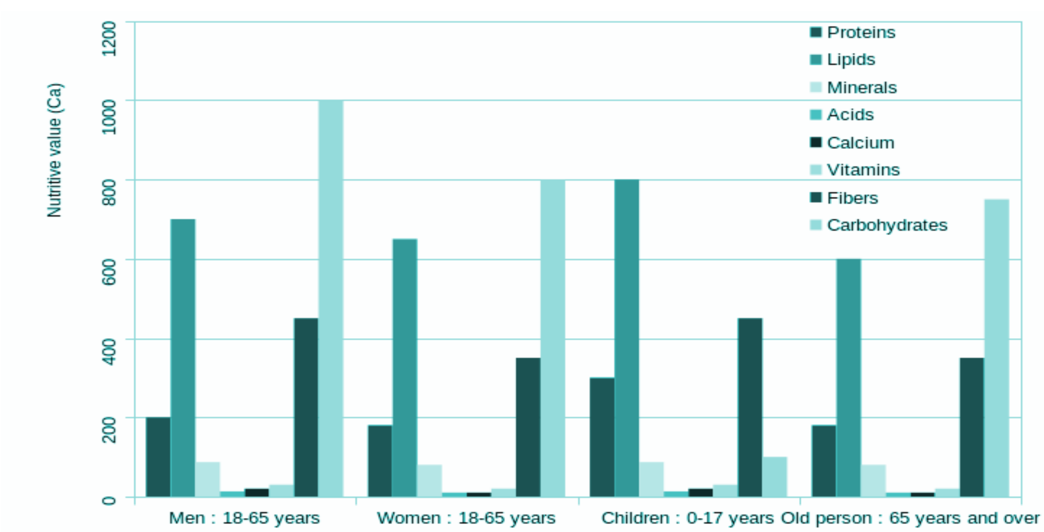

Figure 3: The Necessary National Year consumption By Type of food (Ca).

value consumption by type of food. Considering the categories of population, it is noted that the consumption is very high in carbohydrates and lipids. This means that national production must take this indicator of decision support into account in planning for optimal food production. The deficient consumption of acids and calcium remains minimal, which does not put much emphasis on production measurements.

The optimal results will be obtained using AMPL (A Mathematical Programming Modeling Language) programming and the KNITRO solver. We must test also the Algorithmic implementation under a Matlab programming

language with global optimization toolboxes & Simulink. The results must demonstrate the commutation process.
The expected solutions are the decision indicators on the quantity of production, consumption, monetary and non-monetary yield in relation to the identified project variables. The technical variables of the control show the mastery of poverty in Burundi. This solving problem can be extended to big production area as other country and continent with some complexity and cost considerations because of the big number of states and control variables.

5 Analysis and Modeling local governance practices in Burundi

The Burundian governance integrates strategic, structural, relational, decision-making and informational logics. The processes implemented by municipalities to make and integrate strategic decisions are indeed identified as a critical performance factor. Decision-making involves two major phases: The problem identification is the collection and analysis of information internal and external to the municipality, in order to determine whether the performance is satisfactory and to diagnose the cause of the problems; The solution of the problem, which concerns the development of the alternatives to be considered, as well as the selection and implementation of one of these alternatives.
The following variables mean the dimensional analysis of local governance in order to improve the performance of managment system.

1. **Strategic Variables**: Existence of a community project, Adaptation of the public service offer to needs

2. **Structural variables**: Nature of political structures, Nature of the internal organization

3. **Relational variables**: Nature of relations between elected officials, administrative agents, elected officials and administrative services, services of the municipality, and those of the municipalities members, users and the municipality, elected officials and citizens.

4. **Decision-making variables**: Nature of the decision-making process, Nature of the allocation of human/financial resources.

5. **Informational variables**: Quality of internal information, Quality of information with users.

The dehumanization of the decision-making process is one of the limits of decision-making: the decision-maker must be understood as the "creator", "actor" and "carrier" of the decision. Humanizing the process implies placing the decision-maker at the heart of the decision, and recognizing that he acts according to his cognitive structure. In addition, complex systems involve as for multiple interested parties and actors, it is necessary to consider on the one hand, the stakeholders concerned by the decision-making, and on the other hand, the way in which the actors of the process collaborate.

6 Conclusion and Future perspective

This paper develops the Dynamic Poverty Predictive algorithm for performance improvment of Burundi management system. This Economic Optimization managment System determines the dynamic and optimal control problem when considering the poverty problem and governance dimension.
High runing performance are demonsrated with results giving feasible trajectories and decision support indicators with a robust optimizing of the national monetary yield objective function. The national food production and the necessary annual consumption in terms of calories per person per day are determined. By comparing the annual food production and the required annual consumption, there is an imbalance between different types of food. Proteins, minerals and vitamins produced in Burundi are sufficient when considering their consumption as required by the entire Burundian population. Real production remains in deficit. The lipids, acids, calcium, fibers and carbohydrates produced in Burundi are insufficient for consumption. This negative contribution proves a Burundian food deficit. It is a decision-making indicator for the design and updating of agricultural policy and implementation programs and projects. Investment and economic growth are only possible when food security is mastered. The capital allocated to food investment must be revised upwards. Demographic control is also a relevant indicator to push forward Burundi among the Emerging Countries in 2040.
Challenges are so many in an Innovative Economic Optimization and mangment system. The persepective future is to establish the decision support indicators for all the economic area. Process of setting up a reference center on the incubation of this research results, the Knowledge transfer process to the community, the Process of perpetuating this technology, the Process of predicting the economic future of Burundi, the Innovation at the service of the community.

Ackonwledgement: This paper was developped in ***Laboratoire Universitaire de la Recherche en Modélisation et en Ingéniérie de la Statistique Appliquée "LURMISTA"***, Institute of Applied Statistic, University of Burundi. It is supported by the Government of Burundi through the State Ministry of Education and Scientific Research.

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Received: July 12, 2023; Published: August 7, 2023